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Technical Note

Variation of Nusselt number with flow regimes behind a circular cylinder for Reynolds numbers from 70 to 30 000

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Abstract

The dependence of the Nusselt number in the separated flow behind a circular cylinder to the cross-flow varies greatly with Reynolds number according to the flow regimes, i.e., laminar shedding, wake transition, and shear-layer transition regimes. The Nusselt number at the rear stagnation point, $Nu_r/Re^{0.5}$, increases with Reynolds number in the laminar shedding regime ($Re < 150$) and the shear-layer transition regime ($3000 < Re < 15000$), corresponding to the shortening of the vortex formation region. On the contrary, the Nusselt number, $Nu_r/Re^{0.5}$, decreases with Reynolds number in the regime in which the wake develops to a complex three-dimensional flow ($300 < Re < 1500$), corresponding to the lengthening of the vortex formation region. This distinctive change affects the correlation of the overall Nusselt number with Reynolds number, i.e., the exponent of the Reynolds number has a lower value for $200 < Re < 2000$ than that for $70 < Re < 200$ and $Re > 2000$.

Keywords: Forced convection heat transfer; Circular cylinder; Separated flow; Reynolds number

1. Introduction

The characteristics of the flow past a circular cylinder change with Reynolds number according to the following regimes: occurrence of vortex shedding at $Re \approx 50$, wake transition at $Re \approx 180$, shear layer transition at $Re \approx 10^3$, and boundary layer transition at $Re \approx 3 \times 10^5$, as described in detail by Williamson [1] and Zdravkovich [2], among others. In the same manner as the flow, the characteristics of heat transfer for a circular cylinder are likely to change for each flow regime, because the heat transfer reflects the characteristics of the flow near the surface. Although a large number of studies have been made on the heat transfer from a circular cylinder to the cross-flow, little attention has been given to correlate the heat transfer to the flow, especially in the wake transition and the shear-layer transition regimes. Hilpert [3] measured the heat transfer from a circular cylinder over a wide range of Reynolds numbers from 2 to 2.3×10^5, and presented an empirical correlation between the Reynolds number and the overall Nusselt number. He found discontinuities in the slope of the correlation at $Re = 4$, 40, 4000 and 40 000, which may be related to the change in the flow pattern. However, the reason for the existence of these discontinuities was not clarified. Eckert and Soehngen [4] and Krall and Eckert [5] measured the local heat transfer around a

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Nomenclature

Cp_b	base pressure coefficient
d	diameter of circular cylinder [m] or [mm]
Gr_d	Grashof number based on d
h	heat transfer coefficient [W/m^2 K]
L	length of circular cylinder [m] or [mm]
Nu	local Nusselt number = hd/λ
Nu_m	overall Nusselt number = $\int_S Nu \mathrm{d}S/S$
$\dot{q}$	heat flux [W/m^2]
Re	Reynolds number = $u_0 d/\nu$
r	radius of circular cylinder = $d/2$ [m] or [mm]
S	contributing area to the heat transfer [m^2]
T_0, T_w	freestream temperature, wall temperature [K]
u_0	freestream velocity [m/s]
ϕ	angle from the forward stagnation point of the cylinder [deg]
λ	thermal conductivity of fluid [W/m K]
ν	kinematic viscosity of fluid [m^2/s]

Subscripts

r	rear stagnation point
x, y	streamwise and vertical coordinate: $x = y = 0$, center of the cylinder

circular cylinder for $Re = 23-597$ and for $Re = 10-4610$, respectively, and presented circumferential distributions of the Nusselt number. They suggested that the variation of the Nusselt number distribution was caused by the change in the flow regime, e.g., the change from steady to unsteady flow with the occurrence of the vortex shedding. However, their understanding seems to be incomplete because no evidence was shown to link the variation of the Nusselt number distribution to the flow regimes.

In order to clarify the change in the heat transfer according to the flow regimes, the present authors performed a measurement of the heat transfer in the separated flow behind a circular cylinder, together with that of the vortex formation length. Reynolds number ranged from $Re = 70$ to 30000, which covered the laminar shedding, the wake transition, and the shear-layer transition regimes.

2. Experimental apparatus and data analysis

Five test cylinders, listed in Table 1, were used in order to investigate the heat transfer over a wide range of Reynolds numbers. Test cylinders 1 and 5, shown schematically in Fig. 1, were fabricated from a balsa wood rod, and test cylinders 2–4 (not shown but the structure was similar to cylinders 1 and 5) were fabricated from an acrylic resin pipe. Each cylinder has a semi-circular section, 200 mm in length for cylinders 1–4 and 270 mm in length for cylinder 5, removed from the center of the cylinder. A sheet of stainless-steel foil of 0.01 mm in thickness covered the cylinder surface including the removed section. The surface of the cylinder was heated by applying a direct current to the stainless-steel foil under conditions of constant heat flux. The temperature difference between the heated surface and the freestream was 10–30 °C. The existence of the enclosed air in the removed section significantly reduces the heat conduction loss from the heated surface to the inside of the cylinder. Each cylinder was placed horizontally in the test section of the wind tunnel. The test section for cylinders 1–4 was 400 mm high and 300 mm wide, and that for cylinder 5 was 150 mm high and 400 mm wide. The cylinder 5 was set at the exit of the wind tunnel in order to reduce the blockage effect. Square end plates, 150 mm in length, were attached to both ends of the cylinder 5. The temperature distributions on the side and rear faces of the cylinders were measured using infrared thermograph (TVS-8502, Avio). The surface of the cylinders was coated using black paint in order to enhance the emissivity of the infrared radiation.

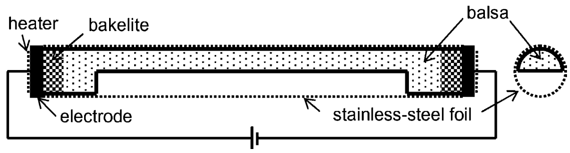

Fig. 1. Schematic diagrams of circular cylinder (cylinders 1 and 5).

Table 1
Circular cylinders

Cylinder	d (mm)	L (mm)	u_0 (m/s)	Reynolds number
1	8.4	300	0.25–16	125–8040
2	10	300	0.5–12	300–7200
3	20	300	2–12	2400–14400
4	40	300	2–12	4800–29100
5	6.4	360	0.17–16	68–6150

Heat transfer coefficient h was evaluated as

$$h = \frac{\dot{q}}{T_w - T_0} = \frac{\dot{q}_{in} - \dot{q}_{rad} - \dot{q}_c - \dot{q}_{sus}}{T_w - T_0}, \quad (1)$$

where $\dot{q}_{in}$ is input heat flux to the stainless-steel foil and $\dot{q}_{rad}$ is the radiative heat flux calculated using the Stefan–Boltzmann equation. The value of the heat conduction from the heated surface to the air inside of the cylinder, $\dot{q}_c$, was estimated by the heat conduction analysis based on the temperature distribution around the cylinder. The circumferential heat conduction through the stainless-steel foil, $\dot{q}_{\text{sus}}$, can be expressed as

$$\dot{q}_{\text{sus}} = -\lambda_{\text{sus}} \frac{1}{r^2} \frac{\mathrm{d}^2 T_{\text{w}}}{\mathrm{d}\phi^2} t_{\text{sus}}, \tag{2}$$

where λ_{sus} and t_{sus} are thermal conductivity and thickness of the stainless-steel foil, respectively. The thermo-physical properties of Reynolds number and Nusselt number were calculated based on film temperature, that is, the mean temperature between the freestream and the heated surface.

The experimental uncertainty of the local Nusselt number for the largest cylinder ($d = 40$ mm) was within 3%, taking into account the uncertainty of the measured temperature, the uncertainty in the estimation of the radiation loss $\dot{q}_{\text{rad}}$, and the uncertainty in the estimation of the heat conduction losses $\dot{q}_{\text{c}}$ and $\dot{q}_{\text{sus}}$. For the smallest cylinder ($d = 6.4$ mm), the uncertainty of the local Nusselt number was ranging from 3% to 10%, corresponding to the freestream velocities of $u_0 = 16$–0.17 m/s. The uncertainty of the overall Nusselt number Nu_{m} was estimated to be within 3%. The effect of natural convection on the overall Nusselt number was negligible because Richardson number, $Ri_d = Gr_d/Re^2$ was 0.09 at maximum in the examined experimental conditions.

3. Results

Fig. 2(a) shows the trend of the Nusselt number at the rear stagnation point, Nu_{r}, along with Reynolds number. The Nusselt number was averaged along span. According to Richardson [6] and Igarashi and Hirata [7], the Nusselt number in the separated flow on the rear of two-dimensional bluff bodies is proportional to the 2/3 power of Reynolds number, which is consistent with the present data for the higher Reynolds numbers ($Re > 1.5 \times 10^4$). However, for the lower Reynolds numbers ($Re < 1.5 \times 10^4$), the trend of the Nusselt number can be expressed as follows:

$$Nu_{\text{r}} = 0.0004 Re^{1.3} (3000 < Re < 15000), \tag{3}$$

$$Nu_{\text{r}} = 2.0 Re^{0.2} (300 < Re < 1500). \tag{4}$$

The above correlations obviously depart from the 2/3 power law. Fig. 2(b) shows the trend of the Nusselt number divided by the square root of Reynolds number, $Nu_{\text{r}}/Re^{0.5}$, the value of which is nearly constant at the forward stagnation point regardless of Reynolds number. The data obtained previously by Schmidt and Wenner [8], Eckert and Soehngen [4], and Krall and Eckert [5] are also plotted. The agreement with the previous data is fairly good. The plots obtained by Eckert and Soehngen and Krall and Eckert have no clear lobe at

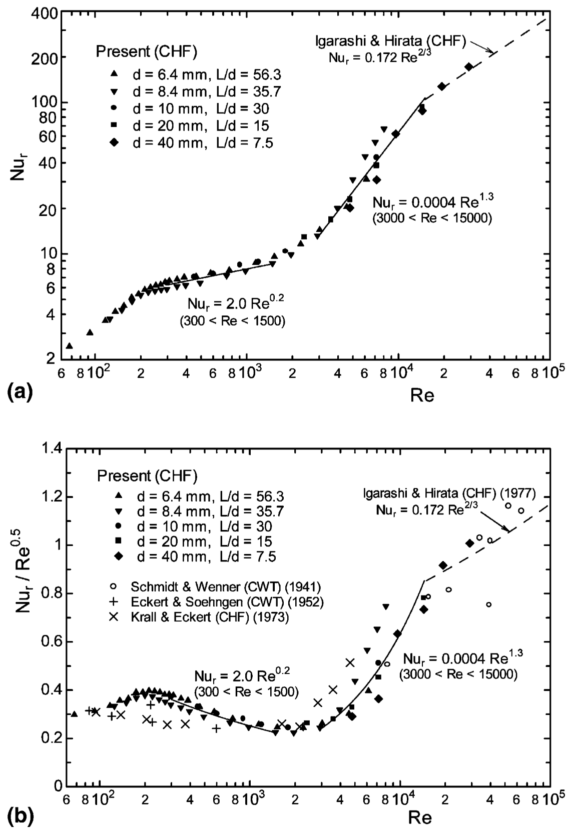

Fig. 2. Nusselt number at rear stagnation point: (a) Nu_{r}; (b) $Nu_{\text{r}}/Re^{0.5}$.

$Re \approx 200$, dissimilar to the present study. It should be noted that the flow velocity could not be measured for the experiment by Eckert and Soehngen, thus Reynolds number was determined using a correlation between the Reynolds number and the overall Nusselt number. This may have led to flattening of the profile. The smaller aspect ratio for Eckert and Soehngen ($L/d = 9$) and for Krall and Eckert ($L/d = 6.3$), which results in the strong three-dimensionality in the flow, may be one of the reasons for the non-clear lobe at $Re \approx 200$. For Reynolds numbers above 3000, the scatter in the Nusselt number is large, probably due to the difference in aspect ratio. However, the trend of the sharp increase over the range of $3000 < Re < 15000$ is consistent.

The Nusselt number at the rear stagnation point, $Nu_{\text{r}}/Re^{0.5}$ has a trend that is similar to that of the "base suction" coefficient, $-Cp_{\text{b}}$ [9,10], and inversely similar to that of the length of the vortex formation region behind the cylinder, as shown in Fig. 3(a) and (b), respectively. The vortex formation length, L_{f}, for the present data was defined as the distance from the center of the cylinder ($x = y = 0$) to the point where the fluctuation of

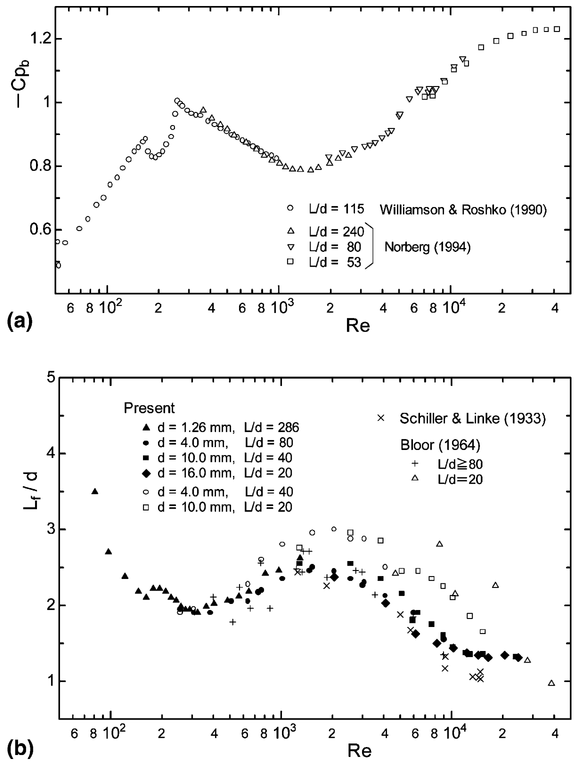

Fig. 3. Flow characteristics behind a circular cylinder: (a) base suction coefficient $-Cp_b$; (b) length of vortex formation region L_f.

velocity (absolute value of the streamwise and vertical velocities) reaches a maximum at the wake center ($y = 0$). The length obtained previously by Schiller and Linke [11] (defined as that to the point of minimum pressure at the wake center) and Bloor [12] (defined as that to the point where fluid from outside the wake first crosses the wake center) are also plotted. In the laminar shedding regime ($50 < Re < 150$) and the shear layer transition regime ($3000 < Re < 15\,000$), the Nusselt number at the rear stagnation point, $Nu_r/Re^{0.5}$, increases with Reynolds number, corresponding to the increase in the base suction coefficient, $-Cp_b$, and the shortening of the vortex formation region. On the contrary, the Nusselt number, $Nu_r/Re^{0.5}$, decreases with Reynolds number in the regime in which the wake develops to a complex three-dimensional flow ($300 < Re < 1500$), corresponding to the decrease in the base suction coefficient and the lengthening of the vortex formation region. For $Re > 15\,000$ (up to the end of the sub-critical regime), in which the vortex formation length is nearly constant regardless of Reynolds number, the Nusselt number increases following the 2/3 power law, with the gradual increase in the base suction coefficient. Namely, the shortening of the vortex formation length leads to the enhancement of the heat transfer at the rear of the cylinder, which demonstrates the close relationship between the flow and the heat transfer in the separated flow region.

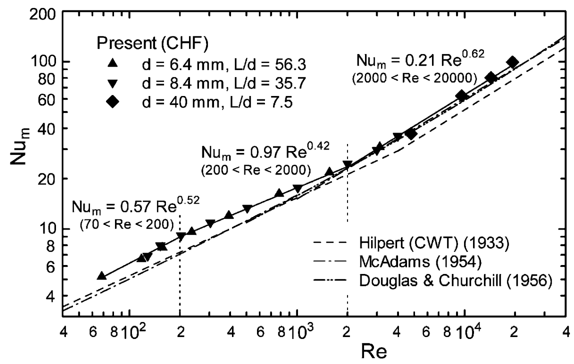

Fig. 4. Overall Nusselt number.

Fig. 4 shows the trend of the overall Nusselt number. For Reynolds numbers above 3000, the present data agree well with the correlations by McAdams [13] and Douglas and Churchill [14]. For Reynolds numbers below 1000, the present data is 10–20% higher than that obtained by Hilpert [3], probably due to the difference in the thermal boundary condition, i.e. the constant wall temperature (CWT) for Hilpert and the constant heat flux (CHF) for the present study. According to the calculations of Krall and Eckert [15], the CHF condition leads to a 15–20% higher overall Nusselt number than that for the CWT condition over the range of $20 \leqslant Re \leqslant 200$, due to a broader distribution of the Nusselt number in the laminar flow region. This difference decreases for higher Reynolds numbers (>3000), because of the sharp increase in the Nusselt number at the rear of the cylinder. Fig. 4 also shows the correlation of the overall Nusselt number for the present data (CHF condition), which can be expressed as follows:

$$Nu_m = 0.57Re^{0.52}(70 < Re < 200), \tag{5}$$

$$Nu_m = 0.97Re^{0.42}(200 < Re < 2000), \tag{6}$$

$$Nu_m = 0.21Re^{0.62}(2000 < Re < 20\,000). \tag{7}$$

It is important to note that the exponent of the Reynolds number for $200 < Re < 2000$ is lower than that for $70 < Re < 200$ and $2000 < Re < 20\,000$. This trend is reflected by that of the local Nusselt number at the rear stagnation point, shown in Fig. 2, which is dominated by the change in the flow regimes behind the cylinder. Interestingly, the similar trend can also be seen in the plot of the overall Nusselt number obtained by Hilpert [3], although not as distinctly. However, this essential feature concerning the correlation of the heat transfer to the flow has been neglected for the most part.

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